

# EXPERIMENTAL ECONOMICS

## STRATEGIC BEHAVIOR WITH BOUNDEDLY-RATIONAL PLAYERS

Ernesto Reuben

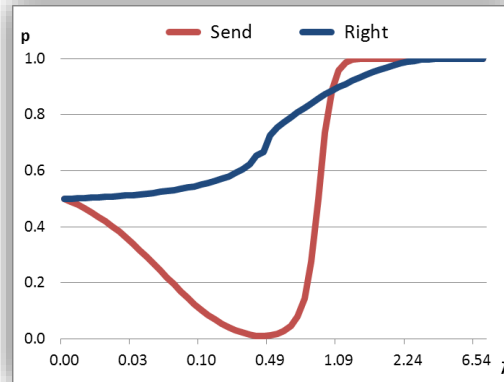
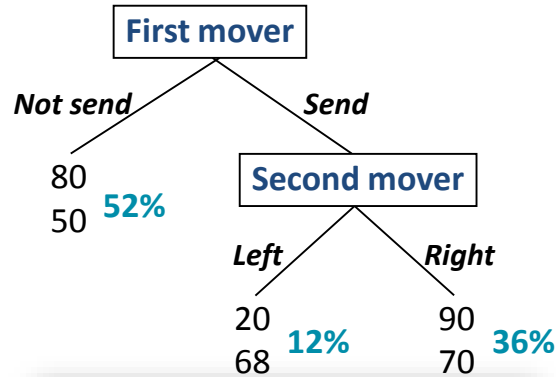
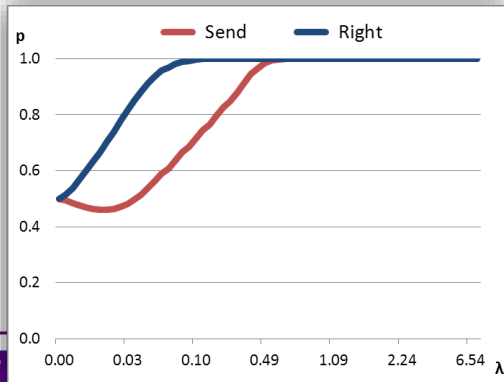
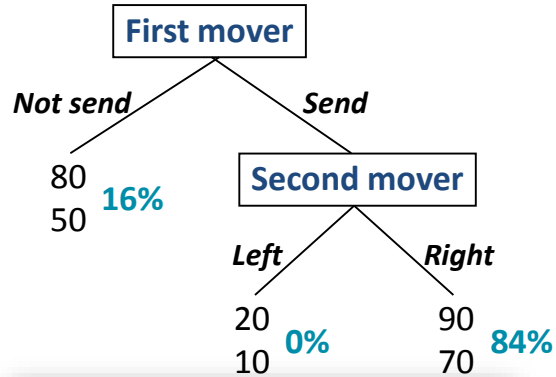
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# QUANTAL RESPONSE EQUILIBRIUM



## How do we model deviations from rationality? (Goeree & Holt 2001)



## Quantal response equilibrium (Goeree et al. 2005)

- Smoothens discontinuous best responses according to a regular quantal response function
- Interiority:**  $P_{is} > 0$  for all  $s$
- Continuity:**  $P_{is}$  is differentiable
- Responsiveness:**  $\partial P_{is} / \partial \pi_{is}$  for all  $s$
- Monotonicity:**  $\pi_{is} > \pi_{ir}$  implies  $P_{is} > P_{ir}$
- Logit quantal response equilibrium

$$P_{is} = \frac{e^{\lambda \pi_{is}}}{\sum_r e^{\lambda \pi_{ir}}}$$

# COGNITIVE HIERARCHIES



How do we model the interaction between different types of boundedly-rational individuals to capture persistent deviations from competitive equilibria?



# THE BEAUTY CONTEST



“It is not a case of choosing those [faces] that, to the best of one’s judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.” –  
*Keynes (1936)*

# THE BEAUTY CONTEST



## The guessing game (Moulin 1986)

- $n$  players simultaneously choose a number  $s \in [1, 100]$
- The winner is the player whose number is the closest to  $\rho \times$  average  $s$  (ties are broken randomly)
  - Standard Nash equilibrium is:
    - Everyone chooses 1 if  $\rho < 1$
    - Everyone chooses 100 if  $\rho > 1$
- The game captures Keynes' intuition and at a basic level some of the incentives in asset markets

# THE BEAUTY CONTEST



SUNDAY MORNING, MAY 14, 1933

SAN ANTONIO EXPRESS

D 9

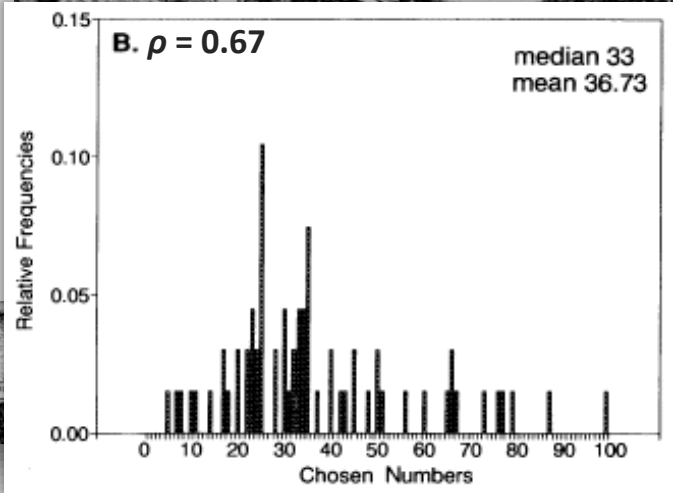
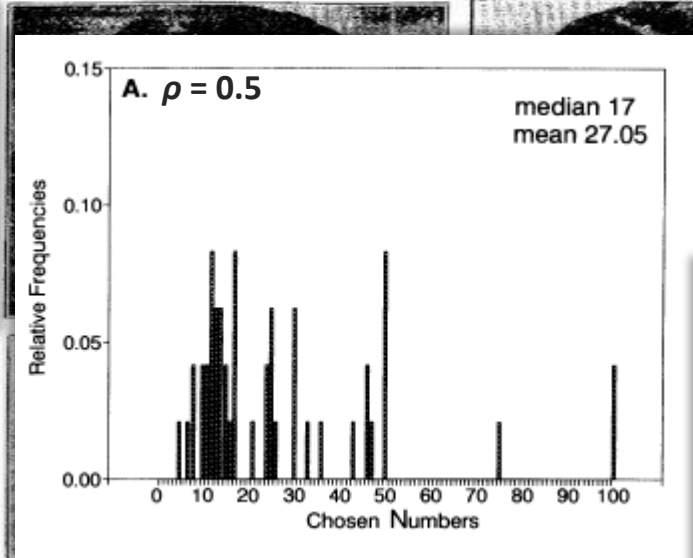
## Here Are Ten More Winners in the Court of Honor Competition

### Nagel (1995)

- Vast majority of numbers above 0 but few dominated strategies



THELMA ELIZABETH MERCHANT



Queen and Ladies-in-Waiting Will Be Announced at

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# THE BEAUTY CONTEST



SUNDAY MORNING, MAY 14, 1933

SAN ANTONIO EXPRESS

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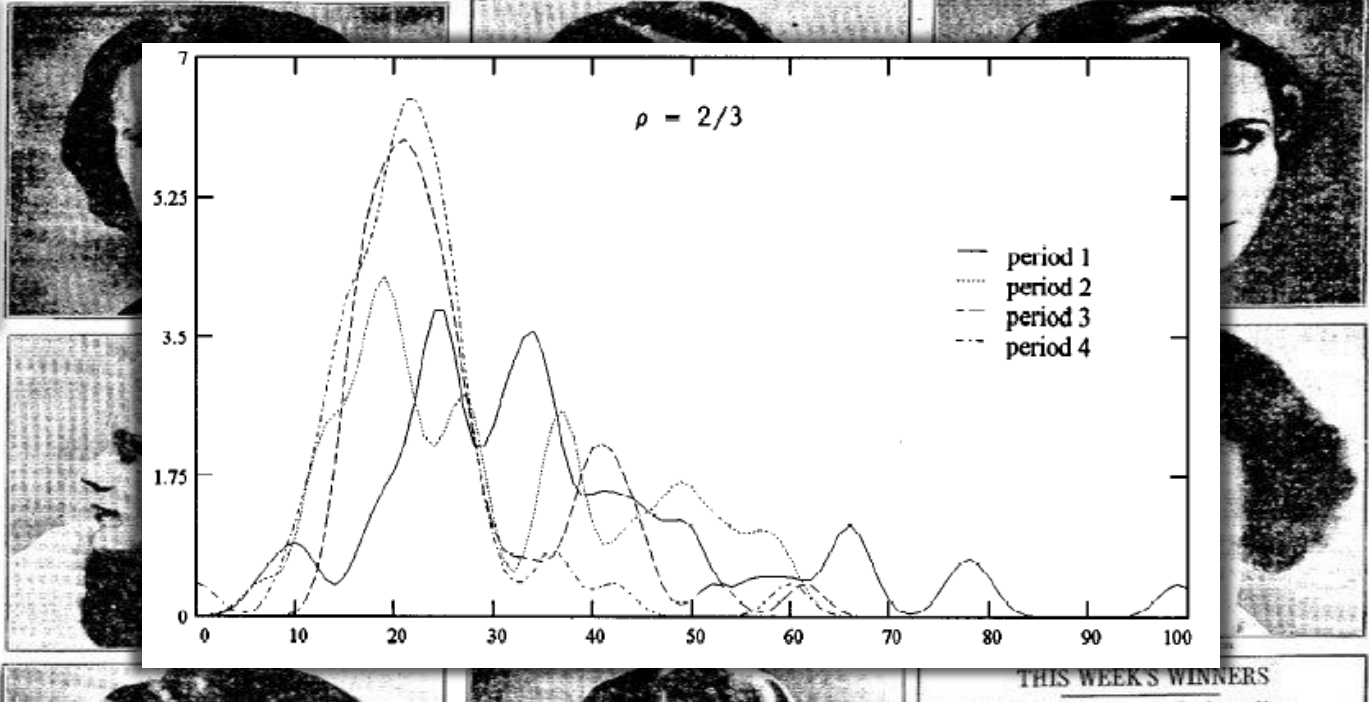
## Here Are Ten More Winners in the Court of Honor Competition

### Nagel (1995)

- Numbers decrease with repetition but very gradually (for high  $\rho$ )



THEODORA ELIZABETH MERRICKSANTL



Queen and Ladies-in-Waiting Will Be Announced at

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# THE BEAUTY CONTEST



SUNDAY MORNING, MAY 14, 1983

SAN ANTONIO EXPRESS

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## Here Are Ten More Winners in the Court of Honor Competition

### Bosch-Domenech et al. (2002)

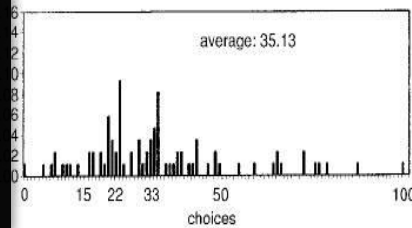
- Robust to many subject pools (e.g., game theorists!)



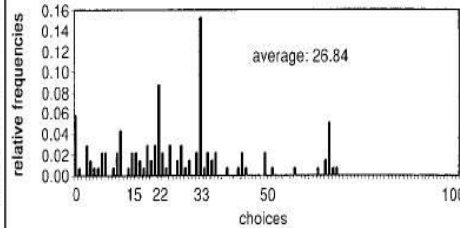
FRANCESCA ELIZABETH MERCHANT

Queen and Ladies-in-Waiting Will Be Announced

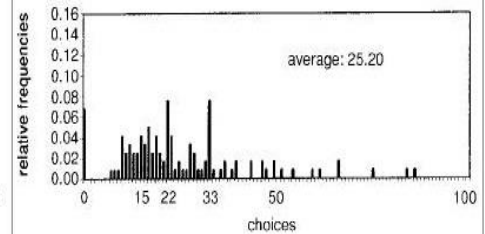
1. Lab experiments (1-5)



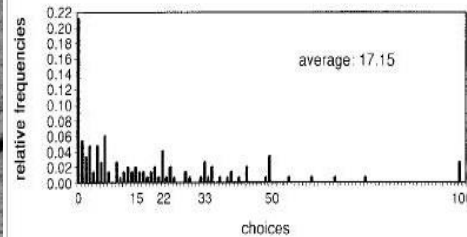
2. Classroom experiments (6,7)



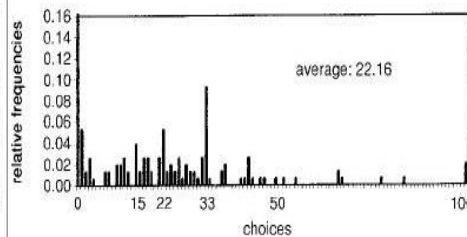
3. Take-home experiments (8,9)



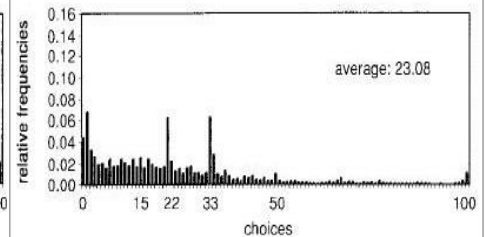
4. Theorists experiments (10-13)



5. Internet Newsgroup experiment



6. Newspaper experiments (15-17)





# COGNITIVE HIERARCHIES



*k*-level  
thinking

- Individuals differ in their **capacity to anticipate the actions of others**, but not in their ability to best respond!
- Assumes individuals hold **overconfident beliefs**

- **Level 0**: chooses randomly (or a default)

Higher levels best respond to their beliefs →  
which are:

- **Level 1**: all others are L0
- **Level 2**:  $p_0$  are L0 and  $p_1$  are L1
- **Level 3**:  $p_0$  are L0,  $p_1$  are L1, and  $p_2$  are L2
- etc. ...



# COGNITIVE HIERARCHIES



*k*-level  
thinking

Prediction for the beauty contest

L0	L1	L2	L3
[1,100]	33.6	22.5	15.1

Stahl & Wilson (1994) and Nagel (1995)

- **Level 0:** chooses randomly between 1 and 100

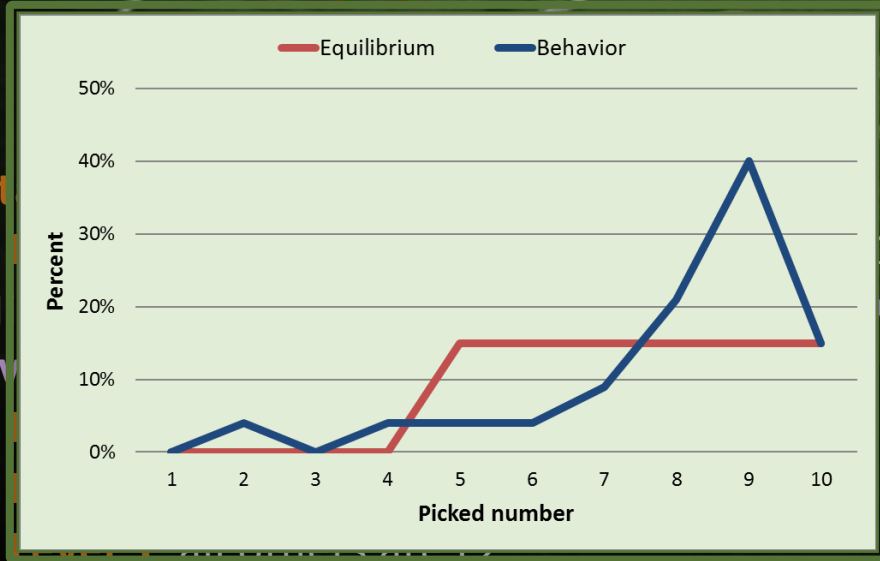
Higher levels best respond to their beliefs →  
everyone is one level below

- **Level 1:** all others are L0
- **Level 2:** all others are L1
- **Level 3:** all others are L2
- etc. ...

# COGNITIVE HIERARCHIES



*k*-level



## The 11-20 game (Arad & Rubinstein 2012)

- 2 players pick an integer number between 11 and 20
- Picking **20** pays **\$20**. **Any other number** pays **\$17** plus **\$20** more if your **number is exactly 1 less** than the other player's

Prediction for the 11-20 game

L0	L1	L2	L3
[11,20]	20	19	18

etc. ...

# COGNITIVE HIERARCHIES



*k*-level  
thinking

Prediction for the beauty contest

L0	L1	L2	L3
[1,100]	33.6	26.9	24.2

Camerer et al. (2004)

- **Level 0:** chooses randomly between 1 and 100
- Higher levels best respond to their beliefs → types follow a Poisson distribution with mean  $\tau$
- **Level 1:** all others are L0
- **Level 2:** 40% are L0 and 60% are L1
- **Level 3:** 28% are L0, 41% are L1, and 31% are L2
- etc. ...

# COGNITIVE HIERARCHIES



**k-level  
thinking**

**Camerer et al. (2004)**

**Higher levels best respond to their beliefs →  
types follow a Poisson distribution with mean  $\tau$**

- By estimating  $\tau$ , we get a measure of the **strategic sophistication of a population in a particular game**

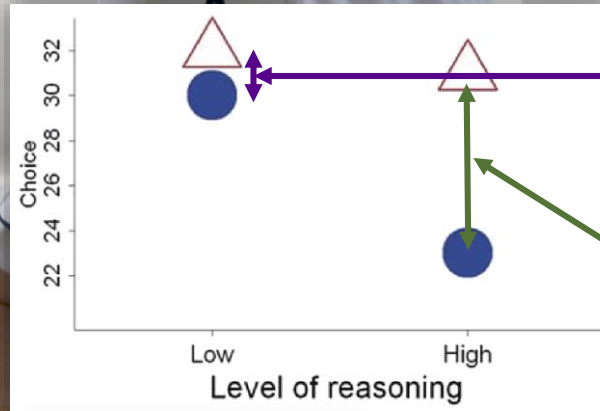
Subject pool or game	Nash equil'm	Data		
		Mean	Mode	$\tau$
$p = 1.1$	200	152.1	150	<b>0.10</b>
$p = 1.3$	200	150.0	150	<b>0.00</b>
$p = 0.9$	0	49.4	50	<b>0.10</b>
$p = 0.7$	0	38.9	35	<b>1.00</b>
1/2 mean	0	26.7	25	<b>1.50</b>
High \$	72	61.0	55	<b>4.90</b>
Low \$	72	54.8	54	<b>2.00</b>
CEOs	0	37.9	33	<b>1.00</b>
German students	0	37.2	25	<b>1.10</b>
80 yr olds	0	37.0	27	<b>1.10</b>
U. S. high school	0	32.5	33	<b>1.60</b>
Portfolio mgrs	0	24.3	22	<b>2.80</b>
Caltech students	0	23.0	35	<b>3.00</b>
Newspaper	0	23.0	1	<b>3.00</b>
Game theorists	0	19.1	0	<b>3.70</b>
Mean				<b>1.30</b>
Median				<b>1.61</b>

# COGNITIVE HIERARCHIES



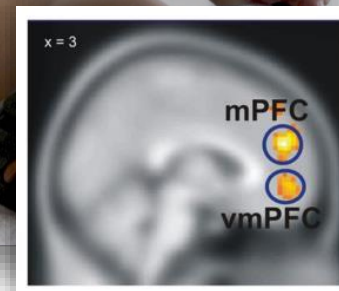
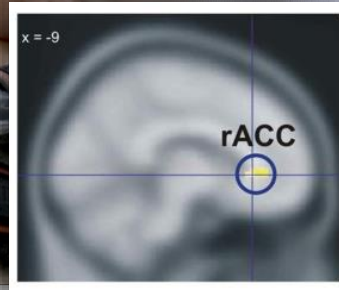
## What are the different levels really capturing? (Coricelli & Nagel 2009)

- fMRI experiment with 20 subjects
- 12 values of  $\rho$  between 0.125 and 1.125
- Play repeatedly vs. **computers** who pick at random and vs. **humans**



Compare play vs. **humans** and **computers**

- **Low level** thinkers
- Rostral anterior cingulate cortex → self-referential
- **Higher level** thinkers
- medial and ventral prefrontal cortex → **theory of mind**



Capacity to put yourself in some else's shoes

# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- Do you think that others think that others are rational?

### Game 1

**Your earnings**

Player 2's actions

	D	E	F
Your actions A	20	14	8
B	16	2	18
C	0	16	16

**Player 2' earnings**

Player 3's actions

	G	H	I
Player 2's actions D	12	16	14
E	8	12	10
F	6	10	8

**Player 3's earnings**

Player 4's actions

	J	K	L
Player 3's actions G	8	20	12
H	0	8	16
I	18	12	6

**Player 4's earnings**

Your actions

	A	B	C
Player 4's actions J	14	18	4
K	20	8	14
L	0	16	18

### Game 2

**Your earnings**

Player 2's actions

	D	E	F
Your actions A	20	14	8
B	16	2	18
C	0	16	16

**Player 2' earnings**

Player 3's actions

	G	H	I
Player 2's actions D	8	12	10
E	6	10	8
F	12	16	14

**Player 3's earnings**

Player 4's actions

	J	K	L
Player 3's actions G	8	20	12
H	0	8	16
I	18	12	6

**Player 4's earnings**

Your actions

	A	B	C
Player 4's actions J	14	18	4
K	20	8	14
L	0	16	18

# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- **R1:** Ignores Player 2's incentives → same choice in both games
- **R2:** Notices that Player 2 has different dominant strategies in the two games → plays A in Game 1 and B in Game 2

Game 1

### Your earnings

Player 2's actions

		Player 2's actions		
		D	E	F
Your actions	A	20	14	8
	B	16	2	18
	C	0	16	16

Game 2

### Your earnings

Player 2's actions

		Player 2's actions		
		D	E	F
Your actions	A	20	14	8
	B	16	2	18
	C	0	16	16



# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- **R1:** Ignores Player 2's incentives → same choice in both games
- **R2:** Notices that Player 2 has the same incentives in both games → same choice in both games

Game 3

**Your earnings**

Player 2's actions

	D	E	F
A	8	20	12
B	0	8	16
C	18	12	6

Game 4

**Your earnings**

Player 2's actions

	D	E	F
A	8	20	12
B	0	8	16
C	18	12	6

# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- **R3:** Notices that Players 2 and 3 have the same incentives in both games → same choice in both games

### Game 3

#### Your earnings

Player 2's actions

	D	E	F
A	8	20	12
B	0	8	16
C	18	12	6

#### Player 2' earnings

Player 3's actions

	G	H	I
D	14	18	4
E	20	8	14
F	0	16	18

### Game 4

#### Your earnings

Player 2's actions

	D	E	F
A	8	20	12
B	0	8	16
C	18	12	6

#### Player 2' earnings

Player 3's actions

	G	H	I
D	14	18	4
E	20	8	14
F	0	16	18

# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- R4:** Notices that Player 4 has different dominant strategies in the two games → anticipates the reaction of Players 2 and 3 and plays A in Game 3 and C in Game 4

Game 3

Your earnings				Player 2' earnings				Player 3's earnings						
				Player 2's actions			Player 3's actions			Player 4's actions				
				D	E	F	G	H	I	J	K	L		
Your actions	A	8	20	12	Player 2's actions	D	14	18	4	Player 3's actions	G	20	14	8
	B	0	8	16		E	20	8	14		H	16	2	18
	C	18	12	6		F	0	16	18		I	0	16	16

Game 4

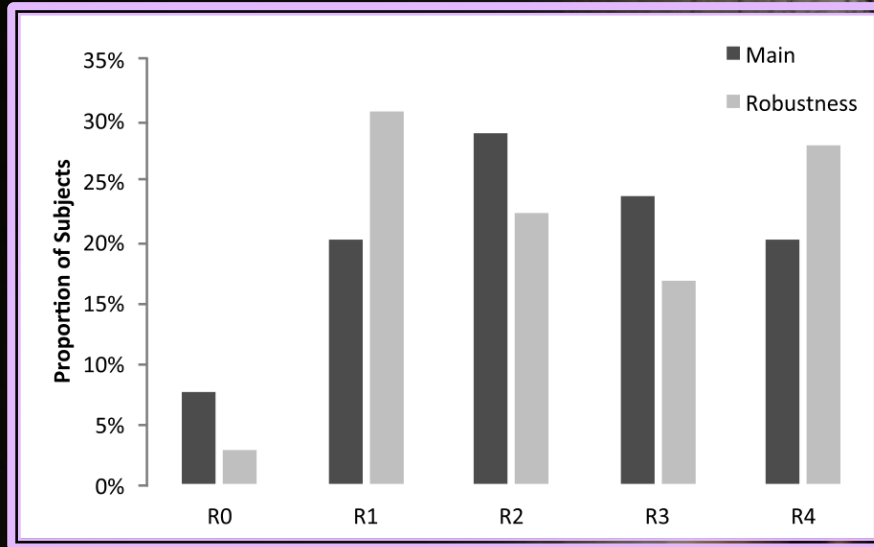
Your earnings				Player 2' earnings				Player 3's earnings						
				Player 2's actions			Player 3's actions			Player 4's actions				
				D	E	F	G	H	I	J	K	L		
Your actions	A	8	20	12	Player 2's actions	D	14	18	4	Player 3's actions	G	20	14	8
	B	0	8	16		E	20	8	14		H	16	2	18
	C	18	12	6		F	0	16	18		I	0	16	16

# HIGHER-ORDER RATIONALITY



## Kneeland (2015)

- **Results:** Fairly even distribution between R1, R2, R3, and R4



# REFERENCES



- Arad, Ayala, and Ariel Rubinstein. 2012. “The 11–20 Money Request Game: A Level- K Reasoning Study.” *American Economic Review* 102 (7): 3561–73.
- Bosch-Domènech, Antoni, José G Montalvo, Rosemarie Nagel, and Albert Satorra. 2002. “One, Two, (Three), Infinity, ... : Newspaper and Lab Beauty-Contest Experiments.” *American Economic Review* 92 (5): 1687–1701.
- Camerer, Colin F, Teck H Ho, and Juin-Kuang Chong. 2004. “A Cognitive Hierarchy Model of Games.” *The Quarterly Journal of Economics* 119 (3): 861–98.
- Camerer, Colin F, T.-H. Ho, and J.-K. Chong. 2004. “A Cognitive Hierarchy Model of Games.” *The Quarterly Journal of Economics* 119 (3): 861–98.
- Goeree, Jacob K, and Charles A Holt. 2001. “Ten Little Treasures of Game Theory and Ten Intuitive Contradictions.” *American Economic Review* 91 (5): 1402–22.
- Goeree, Jacob K, Charles A Holt, and Thomas R Palfrey. 2005. “Regular Quantal Response Equilibrium.” *Experimental Economics* 8 (4): 347–67.
- Kneeland, Terri. 2015. “Identifying Higher-Order Rationality.” *Econometrica* 83 (5): 2065–79.
- Keynes, John M. 1936. *The General Theory of Employment, Interest and Money*. Cambridge: Macmillan Cambridge University Press.
- Moulin, Hervé. 1986. *Game Theory for Social Sciences*. New York, NY: New York Press.

# REFERENCES



- Nagel, Rosemarie. 1995. “Unraveling in Guessing Games: An Experimental Study.” *American Economic Review* 85 (5): 1313–26.
- Stahl, Dale O, and Paul W Wilson. 1994. “Experimental Evidence on Players’ Models of Other Players.” *Journal of Economic Behavior & Organization* 25 (3): 309–27.